Chiral Ribbons Formed by Nematic Elastomer Films

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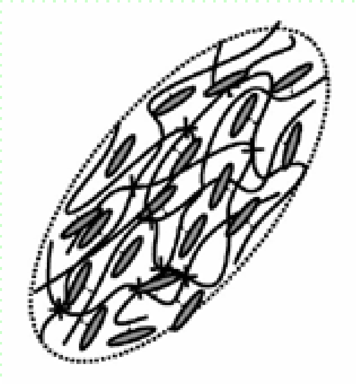
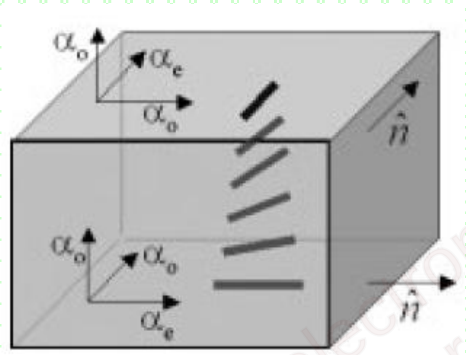
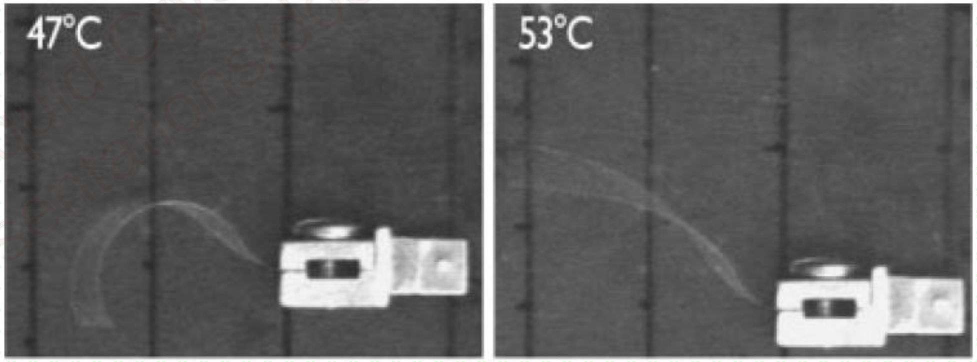
Introduction



combination of elasticity
and orientational order

external stimuli → shape change





G. Mol *et al*, *Adv. Funct. Mater.*, 2005, **15**, 1155-1159

Our goal: to *theoretically* investigate shape change

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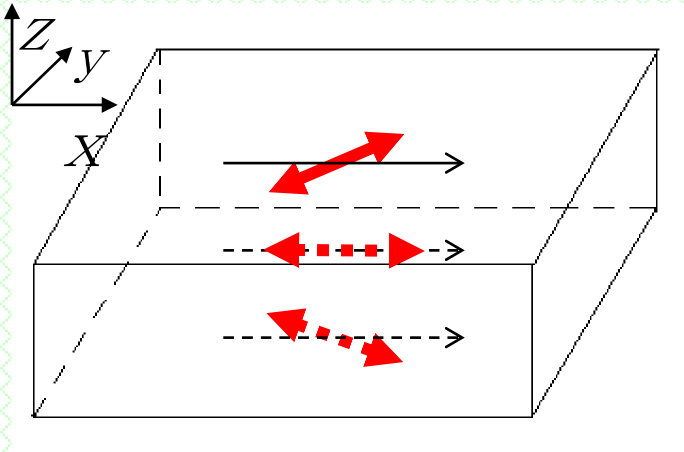
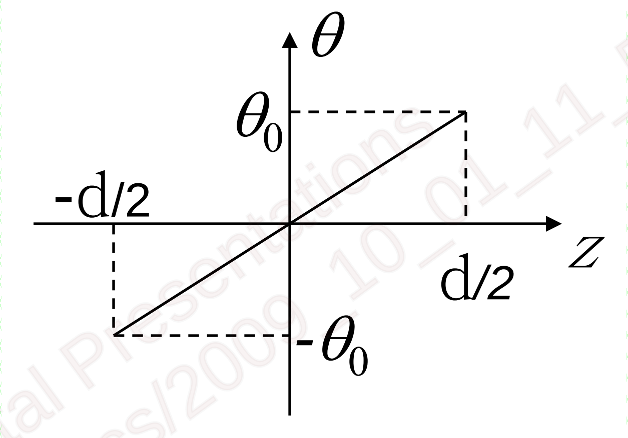
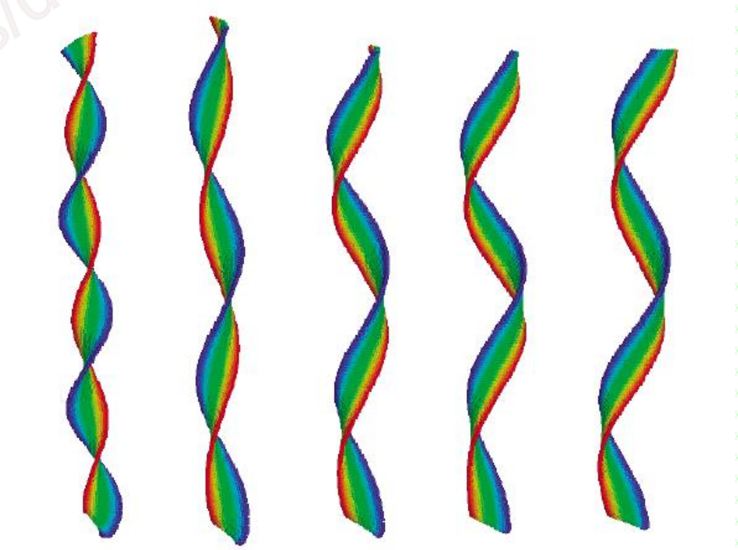
Theoretic Problem





heating
→nematic to isotropic transion
→ expansion/contraction along different axes
→various chiral ribbons



R.L.B. Selinger *et al*, *Phy. Rev. Lett.* **93**, 158103, (2004)

Elastomer Ribbons

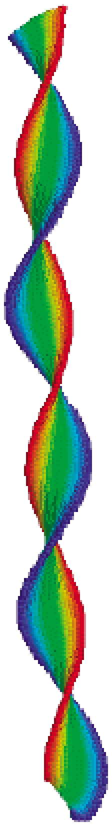
non-isometric → Fopple and von Karman problem

in-plane elastic energy
out-of-plane bending energy
} → shape

L.D. Landau and E.M. Lifshitz, *Theory of Elasticity*

Chiral Ribbons

Helicoids

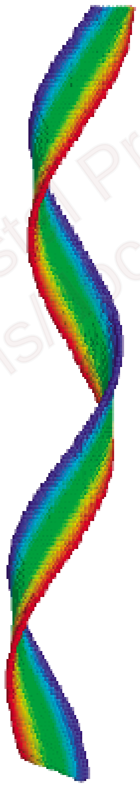


nonzero Gaussian curvature, **zero** bending.

large in-plane stretch energy cost, **no** bending energy cost.

Preferred for **narrow** but **thick** ribbons .

Spiral Ribbons



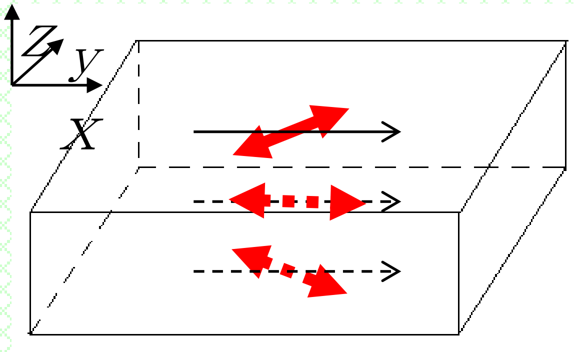
~zero Gaussian curvature, **nonzero** bending.

~no in-plane stretch energy cost, **large** bending energy cost.

Preferred for **wide** but **thin** ribbons .

Elastic Energy

$$f = \mu \, [\mathrm{Tr}\, \underline{\varepsilon}^2 - \alpha \, \mathrm{Tr}\, \underline{\varepsilon}(\underline{Q} - \underline{Q}_0)]$$



Q_0, initial order tensor;

Q, order tensor in body frame vanishes in the isotropic phase.

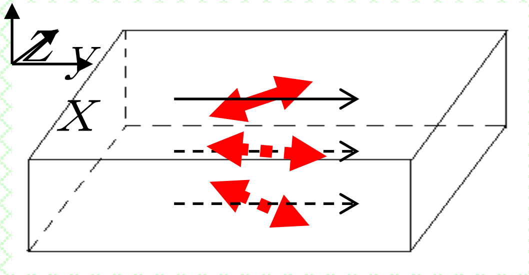
nonlinear strain tensor

$$\epsilon_{ij} = \frac{1}{2}\left(\partial_i u_j + \partial_j u_i + \partial_i u_k \partial_j u_k\right)$$

incompressibility → $\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = 0$

thin film approximation → $\varepsilon_{xz} = \varepsilon_{yz} = 0$

2D Effective Energy



Integrating along Z-direction

chiral symmetry breaking term

$$f_{2d} = a_K K^2 + bC_{xy} + a_{xy}(2C_{xy}^2 + K) + a_{xx}C_{xx}^2 + a_{yy}C_{yy}^2$$

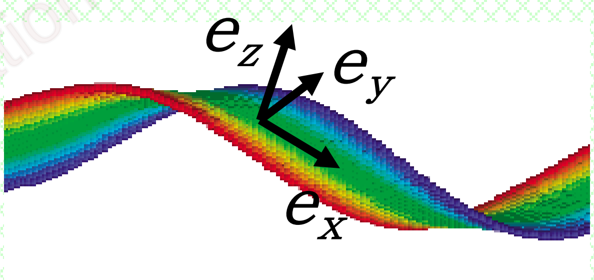
in-plane elastic energy

out-of-plane bending energy

curvature tensor

$$C_{ij} = \partial^2 h / \partial x_i \partial x_j$$

$$K = C_{xx}C_{yy} - C_{xy}^2$$



$a_{xy}>0 \Rightarrow K<0$

R. Ghafouri and R. Bruinsma, *Phy. Rev. Lett.* **94**, 138101 (2005)

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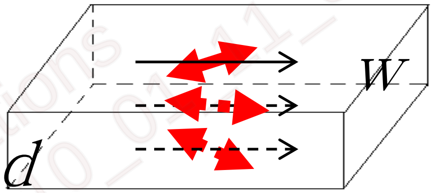
Result

three parameters: $w/d,\ \alpha S,\ \theta_0$

$$f = \mu\,[\mathrm{Tr}\,\underline{\varepsilon}^2 - \alpha\,\mathrm{Tr}\,\underline{\varepsilon}(\underline{Q} - \underline{Q}_0)]$$



$$v = \frac{w}{d}\alpha S \frac{\Lambda_{0y}}{\Lambda_{0x}^{3}} \frac{3(\sin 2\theta_0 - 2\theta_0 \cos 2\theta_0)}{(2\theta_0)^2}$$

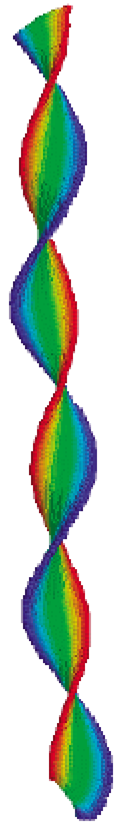
$$r = (\Lambda_{0x} / \Lambda_{0y})^2$$

$$g = \left(\frac{d}{w}\right)^2 \frac{320}{9\Lambda_{0x}^{2}}$$

$$\Lambda_{0x} = \left(1 + \frac{1}{2}\alpha S\left(\frac{1}{3} + \frac{\sin 2\theta_0}{2\theta_0}\right)\right)^{1/2}, \quad \Lambda_{0y} = \left(1 + \frac{1}{2}\alpha S\left(\frac{1}{3} - \frac{\sin 2\theta_0}{2\theta_0}\right)\right)^{1/2}$$

For $v < \sqrt{96g / r}$, or $w / d < g(\alpha S, \theta_0)$

→Helicoids

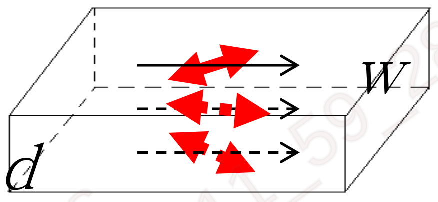


$$C_{xx} = 0$$

$$C_3 = wC_{xy} : \quad v + 2C_3 / r + 4C_3^{\,3} / (r^2 g) = 0$$

Result

For $v > \sqrt{96g / r}$, or $w / d > g(aS, q_0)$

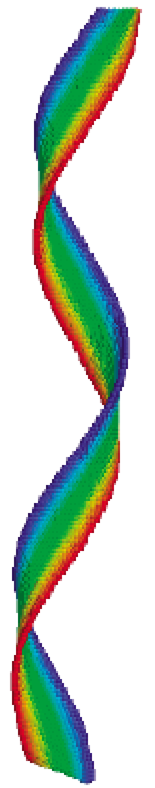


→Spiral Ribons

$$C_{xx} w = \sqrt{v^2 r / 64 - 3g / 2}$$

$$C_{xy} w = vr / 8$$

If $w/d \to \infty, \ C_{xy} \to \sqrt{r}\, C_{xx}, \ K \to 0$



C_{XX} curvature of central line

C_{XY} torsion of central line

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Simulation: Talk TM9

Poster of Gimenez-Pinto