Ordering effects in electric splay Freedericksz transitions

Pierluigi Cesana, SISSA, Trieste, Italy

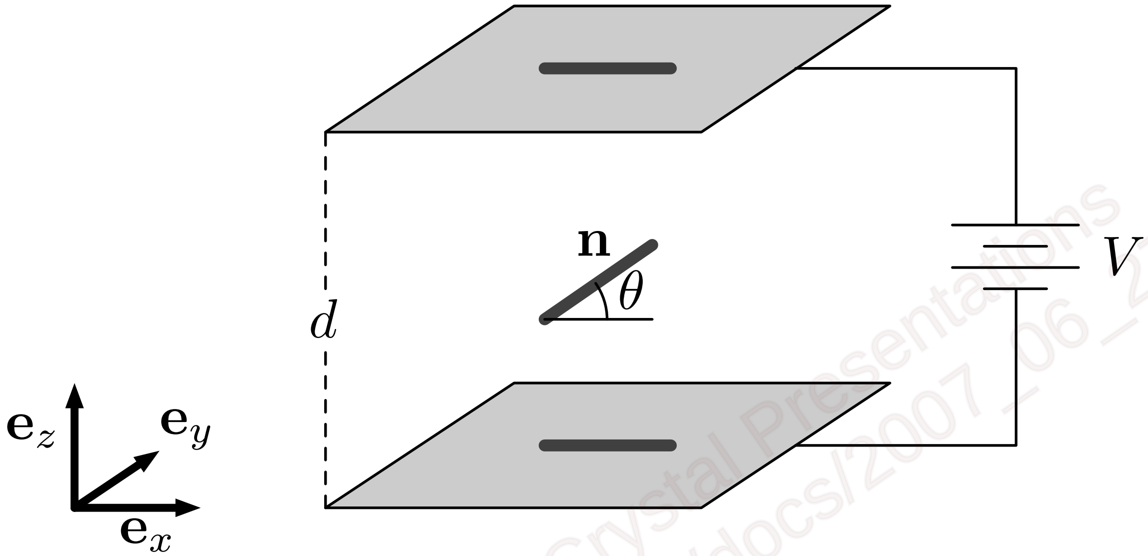
Paolo Biscari, Politecnico di Milano, Italy

Motivation

We aim at investigating the effects of a variable degree of orientation on the equilibrium configurations in electric splay Freedericksz geometry

- Small voltages *disorder* the sample
- Dependence of V_{cr} on s_0
- Most interesting regime: high anisotropy $(\varepsilon_\perp \ll \varepsilon_\parallel)$

Geometry



Dimensionless variables $u = z/d \in [0,1]$, $\phi = \varphi/V$, such that

$$\mathbf{E} = \boldsymbol{\nabla}\varphi = \frac{V}{d}\,\phi'(u)\,\mathbf{e}_z$$

Uniaxial order tensor $\mathbf{Q}(u) = s(u)\big(\mathbf{n}\otimes\mathbf{n} - \frac{1}{3}\mathbf{I}\big)$, with director

$$\mathbf{n}(u) = \cos\theta(u)\,\mathbf{e}_x + \sin\theta(u)\,\mathbf{e}_z$$

Free energy density

$$\Psi = \Psi_{\rm el}(\mathbf{Q}, \nabla\mathbf{Q}) + \Psi_E(\mathbf{Q}) + \Psi_{\rm LdG}(\mathbf{Q})$$

with

$$\Psi_{\rm el} = \tfrac{1}{2}\, K\, |\nabla\mathbf{Q}|^2 = K\left(s^2\, |\nabla\mathbf{n}|^2 + \tfrac{1}{3}\, |\nabla s|^2\right)$$

$$\Psi_E = -\tfrac{1}{2}\, \mathbf{D}\cdot\mathbf{E} = -\tfrac{1}{2}\, (\bar\varepsilon\, \mathbf{E} + \varepsilon_{\rm a}\, \mathbf{Q}\mathbf{E})\cdot\mathbf{E}$$

$$\Psi_{\rm LdG} = A\, {\rm tr}\, \mathbf{Q}^2 - B\, {\rm tr}\, \mathbf{Q}^3 + C\, {\rm tr}\, \mathbf{Q}^4 = \tfrac{2}{3} A\, s^2 - \tfrac{2}{9} B\, s^3 + \tfrac{2}{9} C\, s^4$$

where

$$\bar\varepsilon = \tfrac{1}{3}\big(\varepsilon_\parallel + 2\varepsilon_\perp\big), \qquad \varepsilon_{\rm a} = \varepsilon_\parallel - \varepsilon_\perp > 0, \qquad \gamma_{\rm a} = \frac{\varepsilon_{\rm a}}{3\bar\varepsilon} \in [0,1)$$

Characteristic lengths

$$\xi_{\rm e}^2 = \frac{2Kd^2}{\varepsilon_{\rm a} V^2}\,, \qquad \xi_{\rm n}^2 = \frac{9K}{4C} = 6\eta^2\xi_{\rm e}^2 \qquad (\eta \lesssim 1)$$

Equilibrium equations

Euler-Lagrange + Maxwell equations

$$s\theta'' + 2s'\theta' + \frac{d^2}{\xi_e^2}\,\phi'^2 \sin\theta\cos\theta = 0$$

$$\frac{2}{3}s'' - 2s\theta'^2 - \frac{d^2}{\xi_n^2}\frac{d\sigma}{ds} + \frac{d^2}{\xi_e^2}\phi'^2\left(\sin^2\theta - \frac{1}{3}\right) = 0$$

$$\frac{d}{du}\left(\phi'(1-\gamma_a s) + 3\gamma_a s\phi'\sin^2\theta\right) = 0$$

Boundary conditions

$$\theta(0) = \theta(1) = 0\,, \qquad s'(0) = s'(1) = 0\,, \qquad \phi(0) = 0\,, \quad \phi(1) = 1$$

Results: Freedericksz transition

- Critical voltage $V_{\rm cr} = \sqrt{\frac{2\pi^2 K s_0}{\varepsilon_{\rm a}}}$

  (renormalization of elastic constant)

- Shape of minimizers. Let $V = V_{\rm cr}\left(1+\epsilon^2\right)$

$$\theta(u) = \epsilon A_1 \sin \pi u + o(\epsilon^2)$$

$$s(u) = s_0 - \epsilon^2 \eta^2 \left(B_1 + B_2 \cos 2\pi u\right) + o(\epsilon^2)$$

$$\phi(u) = u + C_1\, \epsilon^2 \sin 2\pi u + o(\epsilon^2)$$

with positive integration constants and, in particular,

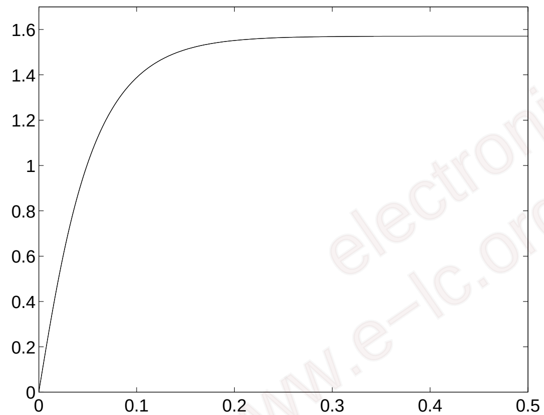
$$A_1^2 = \frac{4(1-\gamma_{\rm a} s_0)}{1+2\gamma_{\rm a} s_0} + O(\eta^2)$$

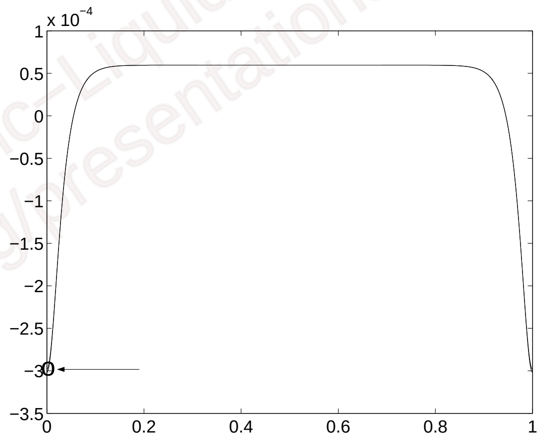
Results: Strong applied voltages

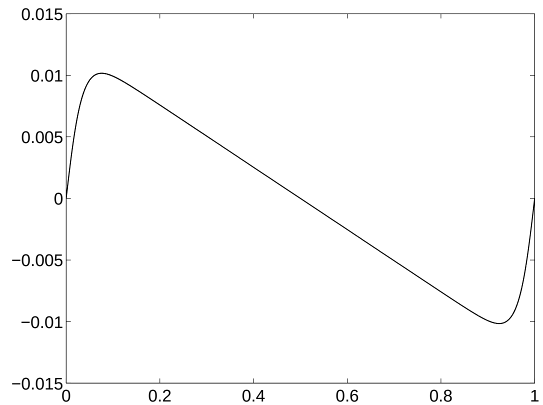
In the limit $V \gg V_{\mathrm{cr}}$ the tilt angle develops a boundary layer to maximize the size of the region where the director is parallel to the external field.

The strong director gradient induce a surface melting

Nematic effects decrease the electric field in the bulk







High anisotropy

Let $\gamma_\mathrm{a} = 1 - \epsilon^2 \quad \left(\varepsilon_\perp/\varepsilon_\| = O(\epsilon^2)\right)$

Singular in Frank limit (Self, Please, Sluckin, 2002)

- Close to the Freedericksz transition the leading orders of the relevant fields are modified: both θ and ϕ become $O(\epsilon^2)$
- For strong applied voltages a double boundary-layer structure arises, and the electric potential develops a finite jump in an infinitesimal layer

Regularized by a finite degree of orientation

- Close to the Freedericksz transition the leading orders are unchanged: $\theta = O(\epsilon)$ while $\phi = O(\epsilon^2)$
- Even for strong fields the electric field remains bounded

Conclusions

- Nontrivial disordered regions/regimes in the presence of an external field
- Regularizing role of a finite degree of orientation in the high anisotropy limit
- From the mathematical point of view, regularization is related to the gain of coercivity in the free energy functional

Comments? Reprints?

cesana@sissa.it